

Relativistic dynamics of quarkonia in strong external fields

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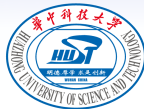
Based on:

Wen et al., PRD 112 (2025);

Zhang et al. to appear;

Li et al. to appear

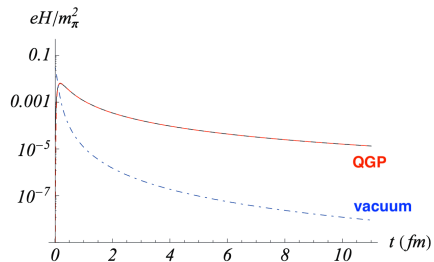
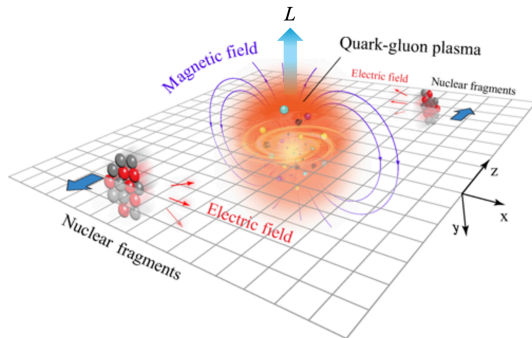
第二届旋转与磁场下的 QCD 物质研讨会
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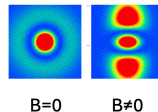
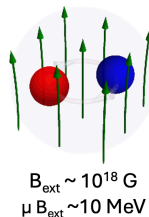
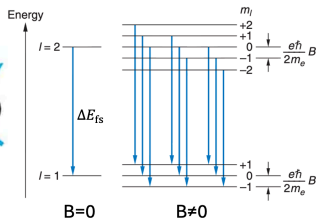
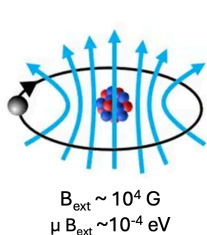


Rotation and magnetic fields in relativistic heavy ion collision

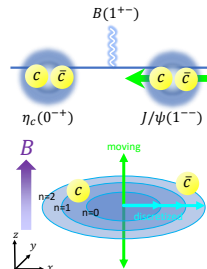
$$L \sim 10^5 \hbar, B \sim 10^{18} \text{ Gauss} \sim 10 m_\pi^2$$

[Deng:2012pc, Tuchin:2013apa]

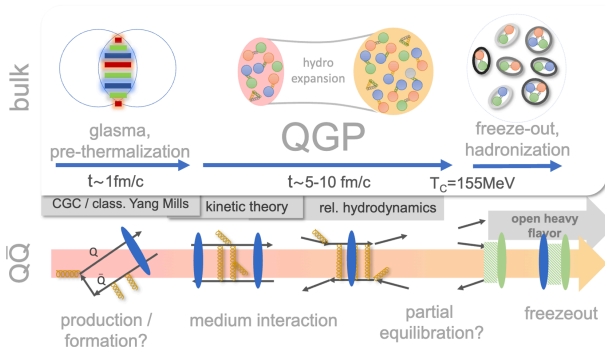
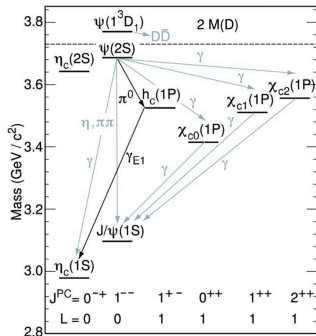




- Atomic physics: nonrelativistic and perturbation
- Hadronic physics: sizeable relativistic and non-perturbative corrections
- Effects in strong magnetic fields:
 - Zeeman effect, Paschen-Back effect, motional Stark effect
 - State mixing, Landau levels, Lorentz deformation, ...
- Effects in strong vortical fields:
 - Barnett effect, Coriolis effect (rotational Zeeman effect)
 - Rotational State Mixing, centrifugal deformation, light cylinder, ...



Quarkonia in heavy-ion collisions



- Quarkonium: the hydrogen atom of QCD
- Ideal probe for hot QCD matter

Theoretical methods: non-relativistic quark model

- Coupling to external magnetic fields $\vec{A}$:

$$\begin{aligned}H_{\text{QCD}} \rightarrow H' &= \sum_i \frac{1}{2m_i} (\vec{p}_i - e_i \vec{A}_i)^2 - \vec{\mu}_{iS} \cdot \vec{B} + \frac{1}{2!} \sum_{i,j} V_{ij}, \\&= H_{\text{QCD}} - \sum_i (\vec{\mu}_{iL} + \vec{\mu}_{iS}) \cdot \vec{B} + \frac{e_i^2}{8m_i} (\vec{B} \times \vec{r}_i)^2\end{aligned}$$

- Dynamics in a rotating frame $\vec{\Omega}$: $\frac{d}{dt}\big|_{\text{lab}} = \frac{d}{dt}\big|_{\text{rot}} + \vec{\Omega} \times$

$$\begin{aligned}H_{\text{QCD}} \rightarrow H' &= \sum_i \frac{1}{2} m_i \vec{v}_i^2 - \sum_i \vec{\Omega} \cdot \vec{S}_i - \sum_i \frac{1}{2} m_i (\vec{\Omega} \times \vec{r}_i)^2 + \frac{1}{2!} \sum_{i,j} V_{ij}, \\&= H_{\text{QCD}} - \vec{J} \cdot \vec{\Omega}\end{aligned}$$

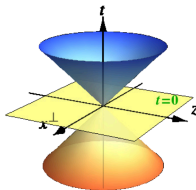
Other approaches: effective Lagrangians, QCD sum rule, Holography, Lattice QCD,

- Full quantum information is contained in the hadronic wave function $|\Psi\rangle$

$$i \frac{\partial}{\partial x^\mu} |\Psi\rangle = P_\mu |\Psi\rangle, \quad (\mu = 0, 1, 2, 3)$$

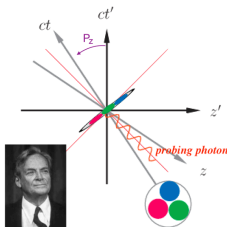
- Relativity allows a free choice of the time variable, e.g. $x^0, x^+ = x^0 + x^3$
- Infinite momentum frame = light-front quantization: retaining the partonic picture

equal-time quantization
 $t = 0$

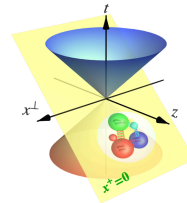


$$i \frac{\partial}{\partial t} |\Psi\rangle = P^0 |\Psi\rangle$$

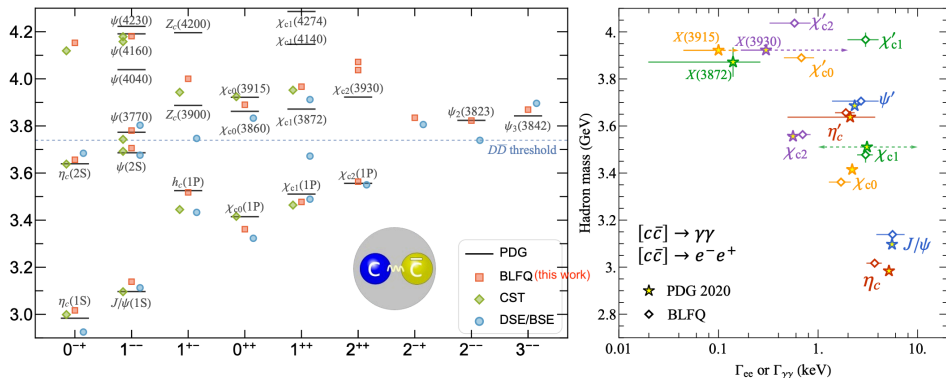
infinite momentum frame
 $P_z \rightarrow \infty$



light-front quantization
 $x^+ = t + z/c = 0$



$$i \frac{\partial}{\partial x^+} |\Psi\rangle = P_+ |\Psi\rangle$$



- Basis light-front quantization: effective Hamiltonian from one-gluon exchange + holographic confinement
- Two free parameters (m_c, κ), rms deviation: 30 MeV
- Good agreement with the PDG data for both the masses and the widths

[Gross:2022hyw]

[LFWFs published on Mendeley Data, doi: 10.17632/cjs4ykv8cv.2]

- Minimal coupling: $\partial^\mu \rightarrow \partial^\mu - ie\mathcal{A}^\mu \Rightarrow \mathcal{L}_{\text{int}} = -J^\mu \mathcal{A}_\mu$
- The light-front Hamiltonian:

$$P^- = P_{\text{qcd}}^- + e \int d^3x J^\mu \mathcal{A}_\mu + \frac{e^2}{2} \int d^3x \bar{\psi} \gamma^\mu \mathcal{A}_\mu \frac{\gamma^+}{i\partial^+} \gamma^\nu \mathcal{A}_\nu \psi, \\ + \frac{e^2}{2} \int d^3x \bar{\psi} \gamma^\mu \mathcal{A}_\mu \frac{\gamma^+}{i\partial^+} \gamma^\nu A_\nu \psi + \frac{e^2}{2} \int d^3x \bar{\psi} \gamma^\mu A_\mu \frac{\gamma^+}{i\partial^+} \gamma^\nu \mathcal{A}_\nu \psi.$$

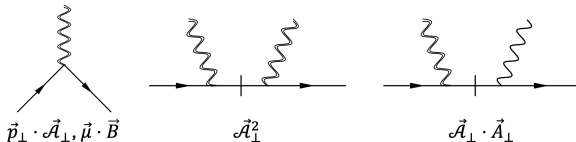
Gauge fixing:
 $\mathcal{A}^+ = 0,$
 $\vec{\mathcal{A}} = -\frac{1}{2}\vec{r} \times \vec{B}$

Extra terms in P^- : the Seagull term, and the terms that modifies the quark potential.

- Exact quantum many-body form:

$$P^- = \sum_i \frac{(\vec{p}_{i\perp} - q_i \vec{\mathcal{A}}_\perp)^2 + m_i^2}{p_i^+} - \vec{\mu}_i \cdot \vec{B} + V_{\text{QCD}}(B)$$

where, $\vec{\mu}_i = g_S(q_i/p_i^+) \vec{S}_i$ is the light-front magnetic moment.



- Minimal coupling in a general frame:

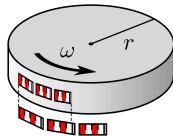
$$S = \int d^4x \sqrt{-g} \mathcal{L},$$

where the metric tensor $g_{\mu\nu}$ becomes,

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \rightarrow g_{\mu\nu} = \begin{pmatrix} 1 - \omega^2 r^2 & 0 & -\omega r^2 & 0 \\ 0 & -1 & 0 & 0 \\ -\omega r^2 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- The Hamiltonian:

$$\begin{aligned} P^- &= \sum_i \frac{p_{i\perp}^2 - (\vec{\Omega} \cdot \vec{J}_i)^2 + m_i^2}{p_i^+} - \vec{\Omega} \cdot \vec{J} + \frac{1}{2!} \sum_{i,j} V_{ij} \\ &= P_{\text{QCD}}^- - \vec{\Omega} \cdot \vec{J} - \sum_i (\vec{\Omega} \cdot \vec{J}_i)^2 \end{aligned}$$



The extra term originates from the length contraction, and is purely relativistic.

One-particle state in external fields

- A pointlike particle in magnetic fields: exact relativistic Landau levels

$$P^- = \frac{\vec{p}_\perp^2 + M_q^2 + \frac{1}{4}e_q^2 B^2 r_\perp^2}{p^+} - (\vec{m} + \vec{\mu}) \cdot \vec{B}$$

where, $\vec{m} = g_L(q/p^+)\vec{L}$ is the orbital magnetic moment. The eigen-energies are,

$$p^- = \frac{M_q^2 + |e_q B|(2n + |m| + m + 1 + 2m_s)}{p^+} \Rightarrow E^2 = p_z^2 + M_q^2 + |e_q B|(2N + 1 + 2m_s)$$

- A pointlike particle in a rotating frame: light cylinder

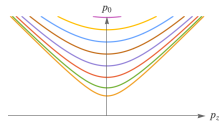
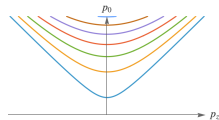
$$p^- = \frac{\vec{p}_\perp^2 - (\Omega m)^2 + M_q^2}{p^+} - \Omega m.$$

The wave functions are

$$\psi_{p_\perp, \ell}(p^+, r_\perp, \theta) = N J_m(p_\perp r_\perp) e^{im\theta},$$

Relativity requires that no particle can travel faster than light, which restricts the space to a light cylinder $r \leq r_\perp^* = \Omega^{-1}$. Then, the transverse momentum $p_\perp$ is quantized:

$$\psi(r_\perp^*) = 0 \quad p_\perp^{(n,m)} = \Omega \xi_{m,n}.$$



Composite particle in magnetic fields

- Wigner representation & max. compatible operator set in vacuum:

$$\{P^\mu, \vec{S}^2, S_z, P, C\} \Leftrightarrow \{\mathcal{M}^2, P^+, H_{\text{cm}}, L_{\text{cm}}, \vec{S}^2, S_z, m_P, C\}$$

Here, m_P is the mirror parity $(x, y, z) \xrightarrow{m_P} (-x, y, z)$.

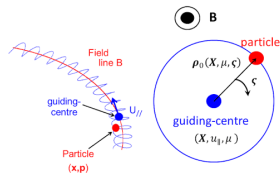
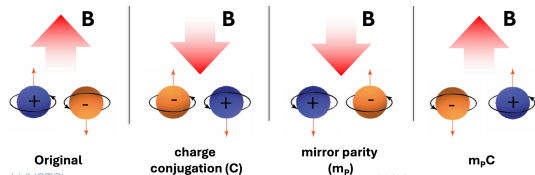
- In magnetic fields:

$$\{\mathcal{M}^2, P^+, \cancel{H_{\text{cm}}}, \cancel{L_{\text{cm}}}, \cancel{\vec{S}^2}, S_z, m_P C\}$$

- Alternative representation in magnetic fields:

$$\{P^-, P^+, \vec{K}_\perp, \cancel{\vec{S}^2}, \cancel{S_z}, m_P C\}$$

where $\vec{K}_\perp = \sum_i \vec{P}_{i\perp} + e_i \vec{A}_{i\perp}$ is the pseudo-momentum, conjugate to the guiding center coordinate.



Charmonium in magnetic fields

- Two-body Hamiltonian:

$$H = H_{\text{qcd}} - q_f e B \left(\frac{m_1 + 2s_1}{x} - \frac{m_2 + 2s_2}{1-x} \right) + \frac{(q_f e B)^2}{4} \left(\frac{r_{1\perp}^2}{x} + \frac{r_{2\perp}^2}{1-x} \right) \equiv H_{\text{qcd}} + H_{\text{int}}$$

- Separating center-of-mass motion and relative motion:

$$H_{\text{int}} = -q_f e B \left(\frac{(1-x)m + 2s_1}{x} - \frac{xm + 2s_2}{1-x} \right) + \frac{(q_f e B)^2}{4} \frac{(1-3x+3x^2)r_{\perp}^2}{x(1-x)} \\ + \frac{(q_f e B)^2}{4x(1-x)} \left(\vec{R}_{\perp}^2 + 2(1-2x)\vec{R}_{\perp} \cdot \vec{r}_{\perp} \right) - q_f e \vec{r}_{\perp} \cdot (\vec{P}_{\perp} \times \vec{B}) + \frac{q_f e}{x(1-x)} \vec{k}_{\perp} \cdot (\vec{R}_{\perp} \times \vec{B})$$

- In non-relativistic limit $x \approx 1/2$, the relative OAM ($m = \vec{r}_{\perp} \times \vec{k}_{\perp}$) does not contribute to the interaction
- Motional Stark effect: electric field $\vec{E}_{\perp} = \vec{V}_{\perp} \times \vec{B}$ induced by c.m. motion where $\vec{V}_{\perp} = \vec{P}_{\perp}/P^+$
- A-B effect: gauge field $\vec{A}_{\perp} = -(1/2)\vec{R}_{\perp} \times \vec{B}$ induced by c.m. motion

- Adopt a basis $\{|N, M\rangle_{\text{cm}}|n, m, l, s_1, s_2\rangle_{\text{rel}}\}$ with basis functions

$$\langle p_1, p_2 | N, M \rangle = \phi_{NM}(\vec{P}_\perp) \phi_{nm}(\vec{k}_\perp / \sqrt{x(1-x)}) \chi_l(x),$$

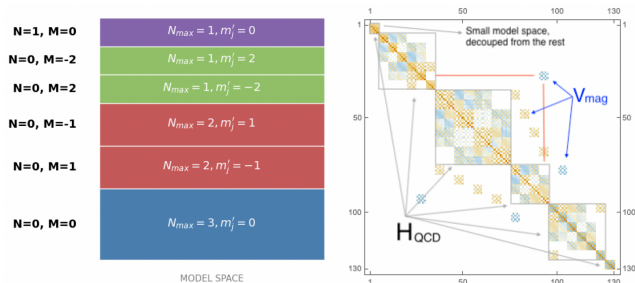
where $P = p_1 + p_2$, $x = p_1^+ / P^+$, $\vec{k}_\perp = \vec{p}_{1\perp} - x\vec{P}_\perp$. ϕ is quantum HO function, and χ is Jacobi polynomials.

- Basis truncations:

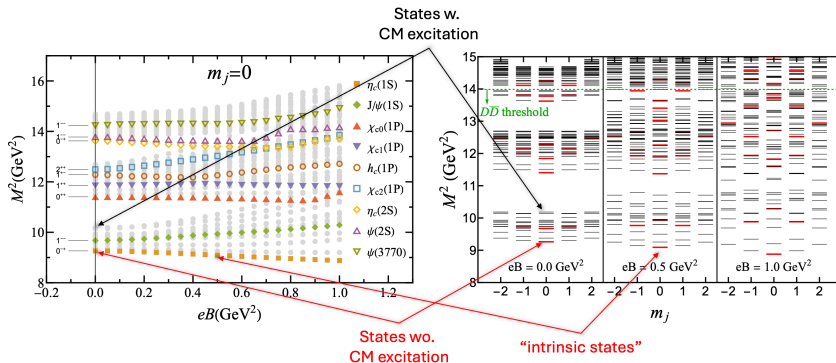
$$2N + |M| + 1 + 2n + |m| + 1 \leq \bar{N}_{\text{max}},$$

$$l \leq L_{\text{max}},$$

$$M + m + s_1 + s_2 = m_j$$

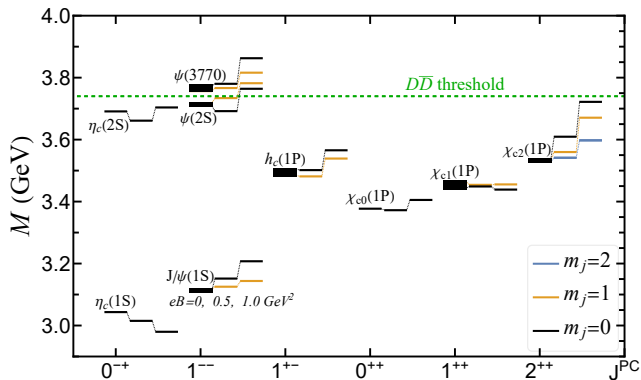


Charmonium spectrum in weak magnetic fields



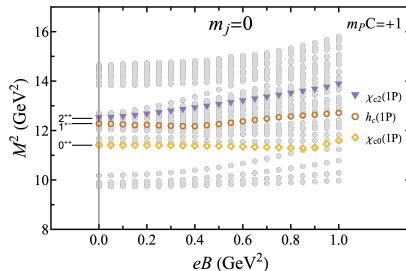
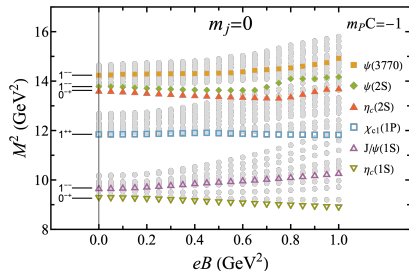
- In vacuum, states with c.m. excitation can be identified by examining $\langle H_{\text{cm}} \rangle$
- In magnetic fields, c.m. motion couples with relative motion
- In weak fields, we define "intrinsic states" as those smoothly connected to the genuine intrinsic states at $eB = 0$

Charmonium spectrum in weak magnetic fields



- Non-linear Zeeman effect: splitting of magnetic projections m_j in magnetic fields
- Level repulsion due to state mixing, e.g. J/ψ - η_c

Charmonium spectrum in magnetic fields

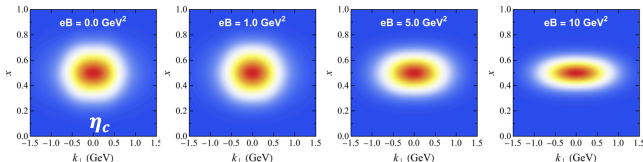


- At strong magnetic fields, states strongly mix and only exact quantum numbers such as $m_P C$ can be used to identify states
- Note that the g.s. of $m_P C = +1$ sector is a J/ψ -like state with c.m. excitation

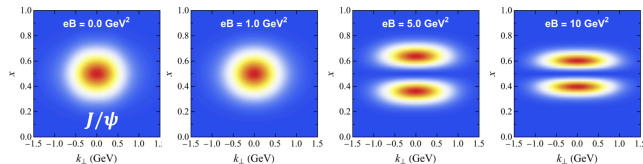
Charmonium density in magnetic fields

$$\rho(x, \vec{k}_\perp) = \sum_{s, \bar{s}} \int \frac{d^2 P_\perp}{(2\pi)^2} \left| \psi_{s\bar{s}}(\vec{P}_\perp, x, \vec{k}_\perp) \right|^2$$

$m_P C = -1$
ground state

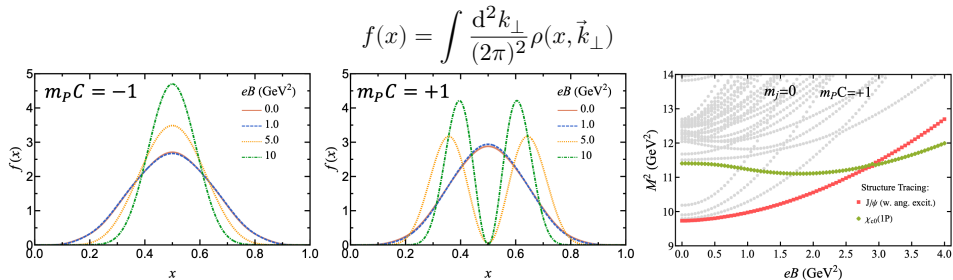


$m_P C = +1$
ground state



- Deformation of η_c ($m_P C = -1$) as the strength of the magnetic field B increases
- Transition of the ground state of $m_P C = +1$ from J/ψ w. OAM to χ_{c0} like state with longitudinal excitation at a magnetic field $B \gtrsim 2.8 \text{ GeV}^2$

Charmonium density in magnetic fields



- Deformation in the longitudinal direction as the strength of the magnetic field B increases
- Transition of a singly peaked distribution to double-hump distribution of the ground state of $m_P C = +1$: consistent with a transition from J/ψ w. OAM to χ_{c0} like state with longitudinal excitation at a magnetic field $B \gtrsim 2.8 \text{ GeV}^2$

Summary and outlooks

- We investigate the properties of charmonium systems in strong external magnetic fields using a relativistic light-front Hamiltonian approach
- Reveal key effects of the magnetic fields: Zeeman effect, spin mixing, deformation, and structural shifts
- Significance of the relativistic corrections and the consistent treatment of the center-of-mass coupling
- Extensions: incorporating the vortical motion of the medium, applications to the spin polarization of baryon and vector meson J/ψ

Thank you!

A scene featuring three Stormtroopers in white armor. The trooper on the left is aiming a blaster upwards, with a red laser beam visible. The trooper in the center is crouching and aiming a blaster. The trooper on the right is aiming a blaster forward, with a bright red light effect at the muzzle. The background is a dark, futuristic interior with blue and red lighting.

I think we're gonna need

Backup